# Time Reversal, Entropy Relativity, and Computational Naturalness: A Unified Perspective on the Arrow of Time

Olav Mitchell Underdal[*†]

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**Abstract**

We investigate the fundamental nature of *time-reversal symmetry* and its progressive breakdown in complex, structured dynamical systems. Drawing on the fluctuation theorem and recent quantum time-reversal experiments, we show that although microscopic laws remain reversible, the emergence of *computational naturalness*, characterized by a compact *calculation cone* for forward prediction and an expansive *retrodiction sphere* for reversal, inevitably yields intrinsic irreversibility. We formalize this via the *naturalness ratio*, comparing the informational requirements of reversal versus prediction, and demonstrate how structural complexity and causal constraints sharply curtail the duration of reversible dynamics. Concurrently, we introduce a formal measure of *entropy relativity*: an *observer-relative entropy* based on a Kullback–Leibler divergence that reproduces Gibbs entropy in equilibrium while capturing nonlocal correlations. Applied to cosmology, entropy relativity reveals that the early Universe, maximally entropic in its co-moving frame, appears low in entropy when measured against its later expanded state, resolving the low-entropy initial condition puzzle. We classify systems into *fully reversible*, *entropy-resistant*, and *structurally irreversible* regimes, and examine the role of *CPT symmetry* in preserving fundamental laws despite quantum-level asymmetries. Together, these insights bridge microscopic reversibility and macroscopic irreversibility, offering a *unified framework* for entropy relativity, computational naturalness, and the cosmological arrow of time.

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## 1 Introduction

The arrow of time is traditionally associated with the increase in entropy as prescribed by the second law of thermodynamics. In this paper, we propose a broader framework in which time-reversal symmetry is not an inherent feature of fundamental laws but is dynamically broken as the universe evolves and organizes itself. We contend that the structural reorganization of complex systems (driven by a principle we call *computational evolution*) naturally aligns with the arrow of time; as systems develop mechanisms to resist local entropy increase, they intrinsically inhibit time-reversal behavior.

[*]E-mail: `olav.mitchell@underdal-law.com`

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### 1.1 Motivation

A central motivation for this study is to integrate thermodynamics, information theory, and complex systems into a unified understanding of the arrow of time. In particular, we address the apparently paradoxical low-entropy state of the early Universe. While classical thermodynamics explains macroscopic irreversibility via the second law, recent quantum advances (including the *fluctuation theorem* and time-reversal simulations) show that microscopic systems can exhibit transient reversibility; however, these insights do not extend to larger, highly structured systems where irreversibility arises not only from entropy production but also from evolved mechanical, electrodynamic, and potential-energy barriers.

Moreover, we reinterpret entropy as a *relative* quantity that depends on system boundaries and reference frames. In cosmology, this perspective illuminates the early Universe's low-entropy appearance: in its own co-moving frame it may have been maximally entropic, yet it appears "low" when measured against a later, expanded spacetime region. Such an apparent paradox can arise if the Big Bang process unfolds over an extended phase (allowing the spatial manifold to expand faster than matter and radiation), so that local equilibrium entropy is hidden behind super-horizon correlations. From this standpoint, two universes evolving in opposite time directions from the same initial state would develop complex structures (for example, biological brains and artificial computers) in a broadly symmetric fashion despite their reversed temporal ordering.

We further extend time-reversal analysis by examining its limitations in systems with advanced computational organization. Examples range from biological neural networks and mechanical clocks to modern computing devices; in each case, intrinsic structural barriers (such as reverse-potential energy wells or voltage potentials) prevent full reversal even without overt entropy production. Building on Crooks [1], Jarzynski [2], and recent quantum time-reversal experiments [4], we integrate these perspectives with novel insights into the structural origins of irreversibility.

### 1.2 Structure of the Paper

This paper is organized as follows:

1. **Review of Time-Reversal Symmetry and Current Knowledge**: Foundational concepts including the fluctuation theorem, quantum time-reversal simulations, and conventional entropy-driven irreversibility.

2. **Entropy Relativity and the Universal Arrow of Time**: Introduction of entropy as an observer-relative measure and its cosmological implications.

3. **Proposed Framework and Hypotheses**: Presentation of *computational evolution* and hypotheses on how structural complexity enforces irreversibility.

4. **Classification of Systems by Time-Reversal Properties**: A taxonomy of physical, biological, and computational systems into fully reversible, entropy-resistant, and structurally irreversible regimes.

5. **Reversibility Bounds**: Derivation of a formal bound on the duration over which reversible dynamics can persist (based on causal volume and structural complexity), showing why high complexity precludes sustained time reversal.

6. **Discussion and Conclusion**: Interpretation of our findings, their broader implications for entropy, structure, and the arrow of time, and suggestions for future research.

## 2 Review of Time-Reversal Symmetry

Time-reversal symmetry is a foundational principle in both classical and quantum mechanics; it signifies that the fundamental equations governing physical dynamics remain invariant when the direction of time is reversed. In an ideal, isolated system, this symmetry implies that if a process is allowed by the laws of physics, its time-reversed counterpart should be equally possible. However, in practice, time-reversal symmetry is often broken by dissipative processes, entropy production, and structural constraints present in complex systems.

This breakdown is intimately linked to the emergence of the arrow of time, as prescribed by the second law of thermodynamics. While the microscopic laws of physics exhibit time-reversal invariance, macroscopic phenomena (such as heat flow and the irreversible processes observed in biological and mechanical systems) display a pronounced asymmetry in time. Understanding the limits and failures of time-reversal symmetry, particularly in nonequilibrium and highly structured systems, is therefore essential for reconciling the reversible nature of fundamental laws with the irreversible behavior we observe in the natural world.

In this section, we review the current state of knowledge regarding time-reversal symmetry, with a focus on the statistical mechanics framework and recent theoretical developments. We begin with a detailed analysis of the *fluctuation theorem*, which provides a quantitative description of entropy fluctuations and the emergence of time-reversal asymmetry in nonequilibrium systems. We then discuss the broader implications of these insights for understanding the breakdown of time-reversal symmetry in complex macroscopic systems.

### 2.1 Fluctuation Theorem

The *fluctuation theorem* (FT) provides a rigorous framework for understanding the statistical properties of entropy production in nonequilibrium thermodynamic systems. It quantifies the probability of observing spontaneous fluctuations that temporarily reduce entropy, thereby offering insights into the underlying time-reversal symmetry at the microscopic level. Foundational studies on the fluctuation theorem include work by Evans et al. (1993) [3], Crooks (1999) [1], and a comprehensive review by Evans and Searles (2002).

Consider a system in thermal contact with a reservoir at temperature $T$. Let $\Sigma_\tau$ denote the total entropy production over a time interval $\tau$. The fluctuation theorem states that the ratio of the probability $P(\Sigma_\tau = A)$ of observing a positive entropy production $A$ to the probability $P(\Sigma_\tau = -A)$ of observing a negative entropy production $-A$ is given by:

$$\frac{P(\Sigma_\tau = A)}{P(\Sigma_\tau = -A)} = e^{A/k_B}, \tag{1}$$

where $k_B$ is the Boltzmann constant.

This relation implies that while transient negative entropy production (i.e., entropy decrease) is possible, such events become exponentially less likely as the magnitude of $A$ increases. In large systems or over extended time intervals, the probability of a significant entropy decrease becomes negligibly small, thereby reinforcing the macroscopic irreversibility described by the second law of thermodynamics.

**Interpretation:** The fluctuation theorem highlights a fundamental asymmetry between forward and reverse processes in nonequilibrium systems. In small systems or over short time scales, fluctuations that momentarily decrease entropy can occur, reflecting the reversible nature of microscopic dynamics. However, as system size or the duration of observation increases, these entropy-decreasing fluctuations become exceedingly rare; hence, the arrow of time dictated by the second law dominates the macroscopic behavior.

**Geometric Motivation:** From a geometric perspective, the fluctuation theorem can be interpreted in terms of phase space trajectories. Each trajectory represents a specific sequence of microstates over time, with a corresponding time-reversed path. The theorem quantifies the asymmetry between the volumes of phase space associated with entropy-decreasing and entropy-increasing trajectories, showing that the former is exponentially smaller. This geometric view underscores how the reversible dynamics at the microscopic level give rise to the irreversible behavior observed in large-scale systems.

## 2.2 Quantum Time-Reversal Simulations

Recent years have witnessed significant progress in simulating time-reversal scenarios within controlled quantum systems. These experiments provide valuable insights into the behavior of time-reversal symmetry at the quantum level, where the microscopic equations of motion are formally reversible; however, practical limitations arise due to decoherence and entropy increase.

One notable approach involves using *quantum computing platforms* (such as superconducting qubits, trapped ions, and nuclear magnetic resonance systems) to implement time-reversal protocols. In these experiments, a quantum system is first evolved forward in time and then subjected to a carefully designed sequence of operations that effectively invert its Hamiltonian, thereby reversing its dynamics [9, 10].

These simulations have demonstrated *temporary reversibility* over short time scales, during which the system's wave function returns to a state close to its initial configuration. However, this reversibility is maintained only in the absence of significant environmental interactions; as soon as *decoherence* or *external noise* is introduced, entropy increases and the time-reversed dynamics rapidly deviate from the ideal trajectory [11, 12].

The breakdown of reversibility in these experiments is closely related to the concept of the *Loschmidt echo*, a measure of the fidelity of time-reversal in quantum systems. The Loschmidt echo quantifies the sensitivity of a system's evolution to small perturbations or environmental interactions, thereby illustrating how even minimal disturbances lead to irreversible behavior in systems governed by time-symmetric equations [13, 14].

**Interpretation and Implications:** These findings underscore the practical limitations of achieving true time-reversal symmetry in quantum systems: successful reversal requires near-perfect control of the Hamiltonian and the suppression of decoherence; as soon as environmental interactions generate entropy, the system swiftly enters an irreversible regime. This behavior reaffirms the fundamental link between entropy production and the emergence of the arrow of time.

Moreover, these results illuminate how time asymmetry arises in macroscopic, structured systems. Environmental coupling dramatically expands the *retrodiction sphere* relative to the *calculation cone*, driving the naturalness ratio $\kappa = I_{\text{rev}}/I_{\text{fwd}}$ well above unity. In such entropy-resistant configurations, reversing dynamics becomes exponentially sensitive to perturbations, showing that time-reversal symmetry is effectively broken once complex interactions dominate.

**Geometric Perspective:** From a geometric viewpoint, the breakdown of time-reversal symmetry can be visualized in terms of phase space trajectories. During forward evolution, a quantum system follows a well-defined path through phase space; under ideal time reversal, this path would be exactly retraced. However, due to perturbations and the accumulation of entropy, the reversed trajectory deviates from the original, thereby highlighting the fundamental asymmetry between forward and reversed processes [16, 17].

### 2.3 The Box Experiment and Entropy Relativity

In the conventional box experiment, a large container of volume $V_{\text{large}}$ is filled with gas particles in thermal equilibrium, thereby achieving its maximal entropy:

$$S_{\text{large}} = k_B \ln W_{\text{large}}, \tag{2}$$

where $W_{\text{large}}$ denotes the number of microstates accessible within $V_{\text{large}}$.

Now, consider compressing the gas into a smaller subregion of volume $V_{\text{small}} \ll V_{\text{large}}$. Traditionally, this configuration is interpreted as a "low-entropy state" because the entropy calculated within the confines of $V_{\text{small}}$ is

$$S_{\text{small}} = k_B \ln W_{\text{small}}, \tag{3}$$

with $W_{\text{small}} \ll W_{\text{large}}$. In other words, compared to the original large volume, the small box has fewer accessible microstates and thus a numerically lower entropy.

However, this comparison can be misleading. If the small subregion is considered as an independent system, its boundaries redefine the constraints and the gas redistributes uniformly until thermal equilibrium is reestablished within $V_{\text{small}}$. In this case, the entropy of the small box becomes

$$S_{\text{small, max}} = k_B \ln W_{\text{small}}, \tag{4}$$

which represents the maximum entropy achievable given its restricted volume. Although $S_{\text{small, max}}$ is numerically lower than $S_{\text{large}}$, it is nonetheless the equilibrium (or maximal) entropy state for the small system.

#### 2.3.1 Self-Similarity Between Large and Small Boxes

This reinterpretation reveals a key point of self-similarity:

- Both the large and the small boxes reach thermal equilibrium within their respective boundaries.
- In each case, the system attains maximal entropy relative to the available number of microstates (i.e., $S_{\text{large}} = k_B \ln W_{\text{large}}$ and $S_{\text{small, max}} = k_B \ln W_{\text{small}}$).

The apparent "low entropy" of the small box arises only when one compares its absolute entropy with that of the larger system; this is simply a consequence of the smaller volume allowing fewer microstates. Viewed in isolation, however, the small box is fully equilibrated, meaning that it has reached the maximal entropy available under its own constraints. This underscores that entropy is a relative measure; its value must be interpreted in the context of the system's boundaries and reference frame.

### 2.4 Cosmological Implications

The relative nature of entropy is especially pertinent in cosmology, where the Universe evolves through epochs characterized by vastly different scales and densities. In conventional Big Bang theory, the early Universe is often depicted as a compressed, high-energy state that is typically described as "low entropy" relative to the present Universe. However, when considered as an independent system, the early Universe should be viewed as maximally entropic within its own co-moving frame; the apparent low-entropy condition arises only when this early state is compared to the later, vastly expanded Universe, particularly under revised dynamical conditions.

If, instead, the conventional picture of a singular, instantaneous blow-up is replaced by a scenario in which the early Universe undergoes a prolonged "burning" phase accompanied by continual cosmic inflation, then the entropy measured from an external, expanded reference frame may appear relatively low. In this interpretation, although the early Universe is maximally entropic within the boundaries of its local (co-moving) frame, it appears as a low-entropy state relative to its later, much larger configuration. We discuss such an alternative model of cosmic inflation in our emerging theory of universal inverse inflation [32].

### 2.5 The Universal Arrow of Time

The universal arrow of time emerges naturally from the interplay between entropy gradients and the dynamics of spacetime expansion. In a modified Big Bang scenario featuring a prolonged burning phase, continual cosmic inflation in the early Universe establishes steep entropy gradients. These gradients may be further accentuated during phases when spacetime expands more rapidly than the matter within it, thereby aligning the Universe along a single temporal direction.

As the Universe evolves:

- **Entropy Gradients**: The conditions established during the early burning phase generate significant entropy gradients, driven by processes such as reheating, gravitational clumping, and the onset of structure formation.
- **Rapid Spacetime Expansion**: The rapid, ongoing expansion introduces new degrees of freedom and dilutes local entropy, even as the total entropy of the Universe continues to increase.
- **Emergent Time Direction**: The combined effects of persistent entropy gradients and continual rapid expansion serve to enforce a single, dominant temporal direction, thus giving rise to the observed arrow of time.

#### 2.5.1 Resolving the Low-Entropy Paradox

This observer-relative perspective, enhanced by a principle of continual inflation, offers a potential resolution to the so-called low-entropy paradox of the early Universe:

- **Local Versus Global Entropy**: Although the early Universe is maximally entropic within its own co-moving frame, its observable entropy appears low when contrasted with the later, expanded state. This apparent paradox results from comparing systems with fundamentally different boundary conditions.

- **Dynamic Evolution of the Early Universe**: A prolonged burning phase combined with rapid expansion, in which space expands faster than matter, may ensure that the early Universe's high local entropy appears as low entropy relative to the reference frame of the expanded Universe, thereby reconciling the early state with the second law of thermodynamics.

**Remark 2.1** (Additional Considerations on Complex Entropy Dynamics)**.** *It should be noted that our discussion thus far has intentionally simplified the multifaceted nature of cosmic entropy evolution. In a more comprehensive treatment, one must consider the transition from a radiation-dominated phase to a regime where matter and gravitational interactions become dominant. This transition involves complex processes, such as the formation of massive objects, the onset of nuclear fusion in stars, and the explosive phenomena of supernovae, that contribute to the reorganization of entropy across various scales. In general, these processes are associated with the formation of localized point sources of energy, which are, by themselves, at or near maximum entropy; yet from the perspective of their cosmic neighborhood, they appear as regions of low entropy. Although these dynamics are critical for a complete understanding of the Universe's thermodynamic history, they lie beyond the scope of our current analysis, which focuses on the relative nature of entropy and the emergence of the arrow of time. A detailed integration of these additional factors would require an expanded theoretical framework, and is left for future work.*

### 2.6 Conclusion

The relativity of entropy provides a unified framework for understanding the thermodynamic history of the Universe and the emergence of the arrow of time. By recognizing that entropy is defined relative to system boundaries and reference frames, and by incorporating additional dynamical processes such as continual inflation, we can reconcile the high local entropy of the early Universe with its apparent low-entropy status when viewed from an external perspective. This approach not only addresses longstanding cosmological paradoxes but also lays the groundwork for exploring the implications of these ideas for time-reversal symmetry and the evolution of macroscopic structures in subsequent sections. Moreover, our perspective suggests that phenomena which appear contradictory on a global scale may be fully consistent when analyzed within their proper local contexts, thereby prompting further investigation into the interplay between dynamical expansion, entropy, and the emergent properties of cosmic evolution.

## 3 Observer-Relative Entropy and Causal Boundaries

Entropy enters the present study in two distinct rôles. Classically it is evaluated over a predetermined volume in phase space, yet in practice any physical observer acquires information only from events within her causal domain. To reconcile these perspectives we introduce an *observer-anchored* entropy that (i) reproduces the standard Gibbs/von Neumann expression when equilibrium is taken as reference, and (ii) quantifies the additional information contained in remote, non-local correlations. The construction below provides the formal bridge required by the naturalness analysis developed in Section 4.

### 3.1 Causal slice and coarse-graining

(a) Let $O$ be an inertial (timelike) observer with world-line $\gamma(\tau)$ and 4-velocity $u_O$.

(b) At proper time $\tau$ define the orthogonal hypersurface

$$\Sigma_\tau \;=\; \big\{p \in \mathcal{M} \;\big|\; (p - O)\cdot u_O = 0 \text{ and } p \text{ lies within the causal region accessible to } O\big\},$$

where $\mathcal{M}$ is the ambient space-time manifold. The boundary of $\Sigma_\tau$ is the relevant horizon: a laboratory wall in terrestrial settings, or the cosmological apparent horizon in Friedmann–Lemaître–Robertson–Walker space-times.

(c) Partition $\Sigma_\tau$ into mesoscopic cells $V_i \subset \Sigma_\tau$ with edge length $\ell$ satisfying $\lambda_{\text{mfp}} \ll \ell \ll \mathcal{R}^{-1/2}$, where $\lambda_{\text{mfp}}$ is the mean free path and $\mathcal{R}$ the scalar curvature. Local thermodynamic equilibrium holds within each cell.

(d) For every $V_i$ let $f_i(\mathbf{z})\, d\mathbf{z}$ be the coarse-grained probability density over micro-states $\mathbf{z}$ compatible with the macroscopic variables $M_i$ that $O$ can in principle measure.

### 3.2 Local entropy density

The Gibbs entropy density of cell $V_i$ is

$$s_i \;=\; -k_B \int f_i(\mathbf{z})\, \ln f_i(\mathbf{z})\, d\mathbf{z}. \tag{5}$$

### 3.3 Observer-relative entropy density

Transport the probability distribution of $O$'s local cell $V_0$ along $u_O$ into the tangent frame of $V_i$; denote the transported distribution by $f_{O\to i}$.

**Definition 3.1** (Observer-relative entropy density)**.**

$$\mathcal{S}_{\text{rel}}(V_i \,|\, O) \;=\; k_B \int f_i(\mathbf{z})\, \ln \frac{f_i(\mathbf{z})}{f_{O\to i}(\mathbf{z})}\, d\mathbf{z}. \tag{6}$$

Expression (6) is a Kullback–Leibler divergence; it measures the information required to specify the micro-state of $V_i$ relative to the prior knowledge encoded in $f_{O\to i}$.

### 3.4 Total observer entropy

Integrating over the observable domain yields the entropy accessible to $O$:

$$S_{\text{obs}}(\tau) \;=\; \sum_i \mathcal{S}_{\text{rel}}(V_i \,|\, O)\, \Delta V_i \;=\; \int_{\Sigma_\tau} \mathcal{S}_{\text{rel}}(x \,|\, O)\, \sqrt{h}\, d^3x, \tag{7}$$

where $h$ is the determinant of the induced three-metric on $\Sigma_\tau$.

### 3.5 Recovery of the classical expression

Select the reference distribution $f_O$ to be the *global-equilibrium prior* $f_*$ that maximises entropy under the conserved quantities of the domain. Since $f_{O\to i} = f_{*,i}$ for all $i$, one may write

$$\mathcal{S}_{\text{rel}}(V_i \,|\, O) \;=\; -k_B \int f_i \ln f_i \, d\mathbf{z} \;-\; k_B \int f_i \ln f_{*,i}\, d\mathbf{z}.$$

The second term depends only on the prior and contributes an additive constant independent of the instantaneous micro-state. Consequently

$$S_{\rm obs}(\tau) \;=\; S_{\rm Gibbs}(\tau) \;+\; \text{const.}, \tag{8}$$

so the observer-relative entropy reproduces the classical total entropy up to an inessential constant offset.

### 3.6 Interpretation and relation to information theory

- If the reference distribution is the equilibrium prior, Eq. (8) ensures correspondence with the usual thermodynamic entropy.
- If $f_{O\to i}$ is taken to be the distribution in $O$'s local cell, $\mathcal{S}_{\rm rel}$ quantifies the additional information contained in remote regions relative to the observer's immediate environment; this includes, for example, correlations arising from large-scale structure formation or outgoing radiation.
- The integral $\sum_i \mathcal{S}_{\rm rel}$ thus measures precisely the information that must be supplied to reconstruct the remote past evolution of the domain. In Section 4 we show that the growth rate of this quantity defines the naturalness density $n(x,t)$ and, after spatial coarse-graining, the global naturalness index $\mathcal{N}(t)$.

**Remark 3.1** (Scope)**.** *The present treatment is mesoscopic and special-relativistic. Extensions to general-relativistic horizon thermodynamics—including Bekenstein–Hawking entropy and the entropy of dynamical trapping horizons—will be addressed in a separate publication (cf. [35]).*

### 3.7 Recent Research in Observer-Dependent Entropy and the Past Hypothesis

The last few years have seen a surge of work that re-examines the *Past Hypothesis* (PH) through the prism of relative or observer-dependent entropy. Rather than postulating a mysteriously low Boltzmann entropy at $t = 0$, these approaches ask: *whose* entropy, relative to *which* reference state, and with respect to *which* algebra of observables? Below we review the strands most relevant to our claim that the pre-inflation Universe was already in (quasi) maximum entropy in its *own* comoving frame, so no special PH is required. Throughout, we contrast each line of research with our *observer-relative KL* measure and *computational-naturalness* account of irreversibility.

#### 3.7.1 Quantum–Reference–Frame Algebra and Gravitational Entropy

**Crossed-product entropy.** DeVuyst *etal.* construct a Type-II vonNeumann algebra by adjoining an *observer clock* to the usual Type-III algebra of gravitational constraints, thereby obtaining a finite von Neumann entropy that varies with the chosen quantum reference frame (QRF) [36]. In their formalism a single global state $\rho$ can have many entropy values

$$S_{(\rm QRF)}(\rho) = -\,\mathrm{Tr}(\rho_{\rm QRF} \ln \rho_{\rm QRF})$$

depending on how observables are dressed. This dovetails with our Eq. (2.4): the KL-based observer-relative entropy reduces to $S_{(\rm QRF)}$ when the prior is taken to be the QRF's equilibrium state. Hence, what looks like a "low-entropy Big Bang" in the global Wheeler–DeWitt frame may already be near-maximal relative to an internal comoving clock.

**Generalised Second Law without PH.** Faulkner–Speranza [37] and Ali–Suneeta [38] prove global and *local* Generalised Second-Law theorems by showing that crossed-product entropy is monotonic under half-sided modular flow, $\Delta S_{\rm gen} \geq 0$, with no Past-Hypothesis assumption beyond the semiclassical vacuum. The arrow of time thus emerges from the *relativity* of entropy to an observer algebra—precisely the principle we employ to dissolve the low-entropy-past puzzle.

### 3.7.2 Relativistic Fluctuation Theorems and Frame-Dependent Irreversibility

Basso, Maziero & Céleri derive a quantum detailed FT in curved spacetime; curvature generates an entropy production term $\Sigma_{\mathcal{O}} \neq 0$ that depends on the observer's trajectory [39]. Cai, Wang & Zhao generalise Crooks and Jarzynski equalities in a covariant stochastic framework, but only *after* fixing an observer's time-reversal map [40]. Together these results show that the probability of "entropy-reducing" histories is exponentially suppressed *relative to a frame*.

Our *calculation-cone / retrodiction-sphere* asymmetry makes this suppression quantitative via the naturalness ratio $\kappa \gg 1$, providing an information-theoretic underpinning to the same observer-relative FT.

### 3.7.3 Entropic Gravity from Quantum Relative Entropy

Bianconi proposes an "entropic action" $I_{\rm grav} = S_{\rm vN}(\widetilde{g}_{\mu\nu} \,\|\, g_{\mu\nu})$ between the matter-induced metric $\widetilde{g}$ and the dynamical metric $g$ [41]. Einstein's equations follow from $\delta I_{\rm grav} = 0$ without inserting a special low-entropy boundary condition. A sequel shows that the same functional yields the Bekenstein–Hawking area law for Schwarzschild black holes [42]. In our framework this corresponds to choosing the *vacuum metric* as reference; the pre-inflation state is then naturally quasi-maximal in entropy, so no Past Hypothesis is required.

### 3.7.4 Entanglement Past Hypothesis and its Observer Dependence

Al-Khalili & Chen recast PH as an *Entanglement Past Hypothesis* (EPH), positing a globally low *entanglement entropy* at $t = 0$ [43]. EPH retains a special boundary condition but shifts the question to "low relative to which Hilbert-space factorisation?". Given the ambiguity of subsystem splits in quantum gravity, EPH implicitly introduces an observer-choice—aligning with our insistence that entropy statements are frame-bound. Our conclusion goes further: once one adopts the comoving QRF, the early state already *saturates* the available entanglement bound, so even EPH becomes unnecessary.

### 3.7.5 Coarse-Grained (Observational) Entropy in Mechanical Cosmology

Scharnhorst & Aguirre model an expanding box of self-gravitating particles and compute time-dependent *observational entropies* [44]. Rapid expansion keeps the matter subsystem near local equilibrium (high entropy) while global gravitational constraints drive the *total* coarse-grained entropy far below its static maximum. Their toy cosmology numerically realises our qualitative claim: what appears "fine-tuned" in global variables is in fact typical once gravitational degrees of freedom and the observer's causal horizon are included.

**Comparison to Our Framework.**

1. **Observer relativity**: All strands reviewed confirm that entropy is meaningful only after specifying an observer, a QRF, or a coarse-graining. Our KL-based entropy [Eq. (2.3)] is the minimal information needed to map one observer's prior into another's posterior, unifying these perspectives.

2. **Past-Hypothesis redundancy**: Whereas EPH and observational-entropy models still impose a boundary constraint, the crossed-product and entropic-action programmes—and our own calculation-cone argument—*derive* the arrow of time without any ad hoc low-entropy postulate: the early Universe is already quasi-maximal in its comoving frame.

3. **Quantitative irreversibility**: Fluctuation-theorem results set probabilistic bounds on entropy-decreasing trajectories; our naturalness ratio $\kappa = I_{\rm rev}/I_{\rm fwd}$ converts these bounds into a concrete measure of computational improbability, explaining why macroscopic time-reversal fails even when micro-reversal is allowed.

Overall, the recent literature converges on the view that "low entropy" is a *relative* statement; once the correct observer algebra is chosen, the initial state need not be specially fine-tuned. Our framework embeds this insight in a single information-theoretic metric that simultaneously explains irreversibility and neutralises the Past Hypothesis.

# 4 Forward Evolution and Reverse Causality from Entropic Naturalness

In this section we *expand upon* the asymmetry between forward and reverse time evolution by analysing the naturalness of initial conditions and the sharply different informational domains required for faithful computation. Our central thesis is that the Universe begins in a *maximally natural state*: a configuration of low *observer- relative* entropy for which strictly local causal data are sufficient to predict the immediate future. Any putative time reversal, by contrast, would have to assemble an *anti-natural state*[1] whose description entails an extensive hierarchy of non-local correlations. The difference is quantitative and is encapsulated in the naturalness ratio

$$\kappa(x^\mu;\tau) \;=\; \frac{I_{\rm rev}(x^\mu;\tau)}{I_{\rm fwd}(x^\mu;\tau)},$$

where $I_{\rm fwd}$ and $I_{\rm rev}$ are the Kolmogorov complexities introduced in Definitions **3.1–3.3**.

Although the microscopic laws remain time-reversal invariant, forward dynamics spontaneously develop causal and structural organisation. The ensuing redistribution of information drives $\kappa$ away from unity. Once $\kappa \gg 1$, reverse evolution becomes exponentially sensitive to perturbations and, for all practical purposes, unattainable.

This informational imbalance is expressed geometrically by two complementary constructs:

* the *calculation cone* $\mathcal{C}_{\rm fwd}$, the minimal bounded region whose data are needed for forward prediction, and * the *retrodiction sphere* $\mathcal{C}_{\rm rev}$, the much larger region that must be specified to reconstruct the past.

Together these constructs provide both an intuitive picture and a formal measure for macroscopic irreversibility.

[1] Earlier drafts employed the phrase "minimally natural." The present term emphasises that the naturalness ratio $\kappa$ is *greater* than unity in such configurations.

### 4.1 Forward Evolution from a Maximally Natural State

Immediately after the Big Bang, the Universe is nearly homogeneous and locally in thermal equilibrium. For any spacetime event $x^\mu$ the calculation cone $\mathcal{C}_{\text{fwd}}(x^\mu)$ associated with a microscopic time step $\delta t$ has volume

$$\mathrm{Vol}\big(\mathcal{C}_{\text{fwd}}(x^\mu)\big) \;\approx\; \ell_{\text{corr}}^3,$$

where $\ell_{\text{corr}}$ is the relevant correlation length (decoherence scale, mean free path, etc.). Because the requisite information is strictly local, $I_{\text{fwd}} \sim \mathcal{O}(1)$ (in suitable units) and $\kappa(x^\mu; \delta t) \simeq 1$; the state is maximally natural.

### 4.2 Reverse Evolution from an Anti-Natural State

*In stark contrast*, reverse evolution demands the recovery of the anti-natural state that would have existed $\delta t$ earlier. This requires the complete micro-state of the retrodiction sphere $\mathcal{C}_{\text{rev}}(x^\mu)$, satisfying

$$\mathrm{Vol}\big(\mathcal{C}_{\text{rev}}(x^\mu)\big) \;\gg\; \mathrm{Vol}\big(\mathcal{C}_{\text{fwd}}(x^\mu)\big),$$

because the sphere must encompass long-range fields, outgoing radiation, and every correlation generated during forward evolution. Accordingly $I_{\text{rev}} \gg I_{\text{fwd}}$ and $\kappa(x^\mu; \delta t) \gg 1$, marking the configuration as anti-natural.

### 4.3 Thought Experiment: The Stirred Cup of Coffee

To illustrate these concepts more concretely, *consider a cup of coffee into which milk is poured.*

**Forward evolution.** At the initial moment the coffee–milk system is stratified; a maximally natural state on the scale of the cup. Stirring induces local turbulence: prediction over $\delta t \approx 0.1$ s requires only the millimetre-scale data inside $\mathcal{C}_{\text{fwd}}$. A back-of-the-envelope count gives $I_{\text{fwd}} \sim 10^3$ bits to specify velocity, pressure, and concentration fields.

**Reverse evolution.** After $\Delta t \approx 30$ s of stirring, the mixture displays fractal striations and has emitted on the order of $N_\gamma \sim 10^{23}$ infrared photons into the kitchen air. Re-establishing the pristine layering would require (i) the exact micro-state of all eddies in the cup ($\sim 10^{10}$ bits), *and* (ii) the phase and momentum of the escaped photons contained in a $c\Delta t \approx 30$ light-s sphere ($\gtrsim 10^{35}$ bits). Hence

$$\kappa = \frac{I_{\text{rev}}}{I_{\text{fwd}}} \approx \frac{10^{35}}{10^3} \sim 10^{32},$$

vividly demonstrating that the retrodiction sphere dwarfs the calculation cone and that practical time reversal is unattainable.

### 4.4 Implications for Entropy and Causality

The severe mismatch between the compact calculation cone and the expansive retrodiction sphere supplies a rigorous information-theoretic basis for macroscopic irreversibility. Forward dynamics propagate the system using local rules and modest data; reverse dynamics demand the reconstruction of correlations whose aggregated information cost grows super-linearly with structural complexity. The naturalness density $n(x,t) = \partial_\tau \ln \kappa|_{\tau=0}$ captures this growth, and its spatial integral defines the global naturalness index $\mathcal{N}(t)$. As $\mathcal{N}(t)$ increases, the time window within which reversible behaviour can be sustained shrinks correspondingly (see Eq. (3.6)).

**Geometric perspective.** Forward evolution occupies a narrow cone in a space-time diagram, whereas reverse evolution requires a broad sphere encompassing a substantial fraction of the system's past light cone. This geometric disparity $\mathrm{Vol}(\mathcal{C}_{\mathrm{rev}}) \gg \mathrm{Vol}(\mathcal{C}_{\mathrm{fwd}})$ provides an intuitive visual rationale for why reversing high-entropy, structurally complex states is infeasible.

**Conclusion of the section.** The contrast between the limited local information required for forward prediction and the vast non-local data demanded for reverse reconstruction, embodied by $\mathcal{C}_{\mathrm{fwd}}$ and $\mathcal{C}_{\mathrm{rev}}$, supplies a comprehensive explanation of macroscopic irreversibility. Microscopic equations may admit time-reversed solutions, yet the rapid increase of the naturalness ratio $\kappa$, and the concomitant growth of observer-relative entropy, ensures that such solutions remain physically out of reach in all complex systems of practical interest.

# 5 Framework: Naturalness, Computability, and Time Asymmetry

The macroscopic arrow of time, we contend, originates in an asymmetry of *local causal computability.* Forward prediction of a structured system can be executed with data confined to a compact causal neighbourhood, whereas exact retrodiction generally demands information that is dispersed, unstable, or inaccessible. The present section formalises this intuition, introduces quantitative measures of *computational naturalness*, and derives an explicit bound on the temporal interval over which a system may be reversed without instability.

## 5.1 Naturalness as local computability

For a spacetime point $(x, t)$ and a microscopic time increment $\tau$, let

$$\mathcal{C}_{\mathrm{fwd}}(x, t; \tau)$$

denote the *calculation cone*: the smallest region of space-time whose micro-state suffices to compute the configuration at $(x, t + \tau)$ to a prescribed accuracy $\varepsilon$. We quantify the computational burden of forward prediction by the causal- volume functional

$$\Xi(x, t; \tau) \;:=\; \mathrm{Vol}\big(\mathcal{C}_{\mathrm{fwd}}(x, t; \tau)\big) \quad [\mathrm{m}^3]. \tag{9}$$

A state is said to be *maximally natural* over the interval $\tau$ if $\Xi$ attains its minimal value allowed by fundamental physical constraints (lattice spacing, Compton wavelength, decoherence scale, *etc.*).

Retrodiction over the same interval requires the *retrodiction domain* $\mathcal{C}_{\mathrm{rev}}(x, t; \tau)$. Let $I_{\mathrm{fwd}}$ and $I_{\mathrm{rev}}$ be Kolmogorov-optimal bit-lengths for describing the two domains; the ratio

$$\kappa(x, t; \tau) \;:=\; \frac{I_{\mathrm{rev}}}{I_{\mathrm{fwd}}}$$

is the *naturalness ratio*. Values $\kappa \simeq 1$ indicate computational symmetry; values $\kappa \gg 1$ signal a breakdown of naturalness and the emergence of irreversibility.

## 5.2 Structural complexity and a reversibility bound

Forward and reverse computability are further influenced by the intrinsic *structural complexity* of the system. We introduce a dimensionless complexity factor $C$, normalised so that

$$C = 0 \text{ for an ideal monoatomic gas}, \quad C > 0 \text{ increases with logical depth, feedback-loop rank, or mutual informat}$$

Laboratory fluids and plasmas typically exhibit $C \sim 10^{0\text{–}1}$; biological neural networks and modern microprocessors can reach $C \sim 10^3$.

**Proposition 5.1** (Computational reversibility bound)**.** *Let $\tau_0$ be the characteristic micro-physical time step (e.g. mean collision time). For any structured system obeying a finite signal velocity, the maximum interval over which its micro-state can be reversed without instability satisfies*

$$\tau_{\text{rev}} \;\le\; \frac{\tau_0}{\Xi(x,t;\tau_0)\,[1+C]}. \tag{10}$$

#### 5.2.1 Sketch of the Proof of Proposition 5.1

*Proof Sketch.* Let $\tau_0$ be the minimal microphysical time step (e.g. a collision time) and suppose signals propagate at finite speed $c_{\text{signal}}$. To reverse the state at $(x, t+\tau_0)$, one must collect the microstate over the retrodiction domain of volume $\Xi_{\text{rev}}$. By definition $\Xi_{\text{rev}} \approx c_{\text{signal}}^3\,\tau_{\text{rev}}^3$. Writing $I_{\text{rev}} \sim \rho_I\,\Xi_{\text{rev}}$ (bits per unit volume) and similarly $I_{\text{fwd}} \sim \rho_I\,\Xi_{\text{fwd}} \approx \rho_I\,\Xi(x,t;\tau_0)$, one finds

$$\kappa = \frac{I_{\text{rev}}}{I_{\text{fwd}}} \;\approx\; \frac{\Xi_{\text{rev}}}{\Xi} \;=\; \frac{c_{\text{signal}}^3\,\tau_{\text{rev}}^3}{\Xi(x,t;\tau_0)}.$$

Imposing $\kappa \lesssim 1$ for practical reversibility and solving for $\tau_{\text{rev}}$ then yields $\tau_{\text{rev}} \lesssim [\Xi(x,t;\tau_0)]^{-1/3}\,c_{\text{signal}}^{-1}$.

Finally, incorporating structural overheads (logical depth, feedback loops) via a dimensionless factor $1 + C$ tightens this to

$$\tau_{\text{rev}} \;\lesssim\; \frac{\tau_0}{\Xi(x,t;\tau_0)\,[1+C]},$$

up to $O(1)$ numerical factors. This establishes Proposition 5.1. □

*Interpretation.* As either the required causal volume $\Xi$ or the complexity $C$ increases, the feasible reversal window contracts. Equation (10) supplies a quantitative analogue of mechanical inertia: high $\Xi$ or high $C$ confers *computational inertia* that resists time reversal.

### 5.3 Differential indicators of naturalness loss

In many practical settings a rapid increase in $\Xi$ is precipitated by large values of the *jerk* field,

$$\mathcal{J}(x,t) \;:=\; \frac{d\mathbf{a}}{dt} = \frac{d^2\mathbf{v}}{dt^2},$$

where $\mathbf{a}$ is the local acceleration. Free-fall motion has $\mathcal{J} \simeq 0$ and hence preserves naturalness. Collisions, turbulent cascades, chemical reactions, and biological computation produce large $|\mathcal{J}|$; the causal domain inflates accordingly, enlarging $\kappa$ and shrinking $\tau_{\text{rev}}$.

### 5.4 Growth rate of the causal domain

For continuous media the temporal growth of $\Xi$ may be expressed in terms of entropy dynamics. Denoting by $\sigma(x,t)$ the local entropy-production density and by $\mathbf{J}_S$ the entropy flux, one finds schematically

$$\partial_t \Xi \;\propto\; \sigma(x,t) - \nabla\cdot\mathbf{J}_S(x,t), \tag{11}$$

so dissipative production and advective removal of entropy serve as *drivers of causal expansion.* Equation (11) links the statistical-thermodynamic language of Section 3 with the computability viewpoint adopted here.

#### 5.4.1 Heuristic Derivation of the Causal Growth Law

Starting from the local entropy-balance equation

$$\partial_t s(x,t) \;=\; \sigma(x,t) \;-\; \nabla\cdot\mathbf{J}_S(x,t),$$

we note that any increase in local entropy requires new degrees of freedom to be tracked within the calculation cone. In a coarse-grained picture, each small volume element $\delta V$ whose entropy grows must be added to $\Xi$. Hence, to leading order,

$$\Delta\Xi \;\propto\; \Delta S_{\text{local}} \;=\; \int_{\delta V} \big[\sigma - \nabla\cdot\mathbf{J}_S\big]\, d^3x.$$

Dividing by $\Delta t$ and taking the continuum limit gives

$$\partial_t \Xi \;\propto\; \int_{\mathcal{C}_{\text{fwd}}} \big[\sigma(x,t) - \nabla\cdot\mathbf{J}_S(x,t)\big]\, d^3x \;\approx\; \sigma(x,t) \;-\; \nabla\cdot\mathbf{J}_S(x,t),$$

where the last step assumes slowly varying fields over the cone. This heuristic links Eq. (11) directly to standard nonequilibrium thermodynamics.

### 5.5 Fluctuations in the Causal Domain

So far we have treated $\Xi(x,t;\tau)$ as a deterministic "volume," but in practice local interactions or external probes can induce *spikes* in the causal region required for forward prediction. To model this, we write

$$\Xi(x,t;\tau) \;=\; \bar{\Xi}(\tau) \;+\; \delta\Xi(x,t;\tau)\,,$$

where $\bar{\Xi}(\tau)$ is the spatial average and $\delta\Xi$ captures local excursions.

**Statistics of spikes.** In a maximally natural state these excursions obey a large-deviation bound

$$\Pr\!\big(\delta\Xi > V\big) \;\lesssim\; e^{-\alpha(V)/k_B}\,,$$

so that severe local expansions occur only with exponentially small probability.

**Implications for reversibility.** One can then distinguish

- the *typical reversibility window* $\tau_{\text{rev}}^{\text{typ}} \sim \tau_0/[\bar{\Xi}(1+C)]$, and
- the *worst-case window* $\tau_{\text{rev}}^{\max} \sim \tau_0/[(\bar{\Xi} + \max \delta\Xi)(1+C)]$,

highlighting that "rare but large" causal-domain spikes can momentarily collapse the interval over which time-reversal remains plausible.

## 5.6 Pure Quantum States, Observer-Relative Entropy, and Maxwell's Demon

To illustrate our framework in a familiar quantum setting, consider first an isolated atom in a pure state:

$$\rho = |\psi\rangle\langle\psi|.$$

Unitary evolution under the Schrödinger equation preserves purity, so the von Neumann entropy

$$S_{\text{vN}}(\rho) = -\text{Tr}\big[\rho \ln \rho\big] = 0$$

remains exactly zero. In our language this implies

$$\mathcal{S}_{\text{rel}} = 0, \quad \Xi_{\text{fwd}} = \Xi_{\text{rev}} \sim O(\ell_{\text{micro}}^3).$$

Both the calculation cone and the retrodiction sphere thereby collapse symmetrically to the atom's microscopic degrees of freedom, apart from quantum-scale spike-like fluctuations in either time direction.

When the atom interacts with an external "demon" $D$, the joint state $\rho_{AD}$ remains pure but the reduced atom state

$$\rho_A = \text{Tr}_D\, \rho_{AD}$$

becomes mixed, so

$$S_{\text{vN}}(\rho_A) > 0.$$

In our framework this shows up as

$$\Xi_{\text{rev}} \gg \Xi_{\text{fwd}}, \quad \kappa = \frac{I_{\text{rev}}}{I_{\text{fwd}}} \gg 1,$$

since retrodiction now requires tracking all atom–demon correlations.

Modeling the demon's measurement by a POVM $\{M_m\}$ enlarges its calculation cone. The subsequent memory erasure (per Landauer's principle) is irreversible and consumes work while producing heat. In our theory this appears as a sudden growth of the retrodiction sphere and the naturalness ratio for the combined atom + demon system.

Thus the simple atom + demon example fits neatly into our picture of calculation cones, retrodiction spheres, and naturalness ratios developed above.

Even so, while this may offer a new interpretive lens on pure and mixed-state quantum evolution, it remains fully consistent with standard Schrödinger dynamics, the von Neumann entropy, and modern analyses of Maxwell's demon.

### 5.7 Summary

Naturalness and computability, rather than entropy flux *per se*, govern macroscopic time asymmetry. Forward evolution is economical: a small calculation cone suffices. Reverse evolution becomes prohibitively expensive once the causal volume $\Xi$ or the structural complexity $C$ grows large, as formalised by the bound (10). Accordingly, although the microscopic equations are time-symmetric, the practical impossibility of supplying the required non-local information renders macroscopic time reversal inoperable.

In the subsequent section we apply these measures to physical, biological, and engineered systems, classifying their time-reversal properties in light of their capacity, or failure, to preserve local causal computability.

## 6 Classification of Systems by Computational Naturalness

The disparity between the calculation cone $\mathcal{C}_{\text{fwd}}$ and the retrodiction sphere $\mathcal{C}_{\text{rev}}$ provides a quantitative yard-stick for macroscopic reversibility. In this section we classify physical, biological, and engineered systems in terms of (i) their microscopic time-reversal behaviour and (ii) the extent to which their evolution can be reconstructed from *local* causal data. Four classes emerge, distinguished by the naturalness ratio $\kappa = I_{\text{rev}}/I_{\text{fwd}}$ and the complexity factor $C$ introduced in Section 5.

Before proceeding we recall the microscopic transformation rules for momentum and spin, for these guarantee that any failure of macroscopic reversal originates in *computability*, not in the fundamental laws.

### 6.1 Fundamental Transformations of Momentum and Spin

A prerequisite for any discussion of macroscopic irreversibility is a clear statement of how microscopic dynamical variables respond to the time-reversal operator $\mathcal{T}$.

Four separate *moments of inertia* must be treated distinctly:

| **Quantity** | **Notation & definition** | **Behaviour under $\mathcal{T}$** |
|---|---|---|
| **Linear momentum** | $\mathbf{p} = m\,\mathbf{v}$ | $\mathbf{p} \longrightarrow -\mathbf{p}$ |
| **Orbital angular momentum** (about an origin) | $\mathbf{L}_{\text{orb}} = \mathbf{r} \times \mathbf{p}$ | $\mathbf{L}_{\text{orb}} \longrightarrow -\mathbf{L}_{\text{orb}}$ |
| **Rotational angular momentum** (rigid body) | $\mathbf{L}_{\text{rot}} = I\,\boldsymbol{\omega}$, $I$ = moment-of-inertia tensor | $\mathbf{L}_{\text{rot}} \longrightarrow -\mathbf{L}_{\text{rot}}$ |
| **Intrinsic quantum spin** | Self-adjoint operator $\mathbf{S}$ | $\mathcal{T}\,\mathbf{S}\,\mathcal{T}^{-1} = -\mathbf{S}$ |

**Physical meaning.**

- Linear momentum reverses because velocities change sign while masses are positive–definite scalars.
- Orbital angular momentum flips with $\mathbf{p}$, the position vector $\mathbf{r}$ remaining unchanged.

- Rotational angular momentum of a rigid body involves the body-fixed angular velocity $\boldsymbol{\omega}$; reversing $\boldsymbol{\omega}$ produces the negative of $\mathbf{L}_{\mathrm{rot}}$.
- Spin undergoes an anti-unitary phase change; although no classical rotation occurs, the expectation values of the spin components change sign, ensuring correct reversal in magnetic fields.

**Combined quantities.** Any linear combination

$$a\,\mathbf{p} \;+\; b\,\mathbf{L}_{\mathrm{orb}} \;+\; c\,\mathbf{L}_{\mathrm{rot}} \;+\; d\,\mathbf{S} \quad (a, b, c, d \in \mathbb{R})$$

transforms to its exact negative. Consequently, *every* fundamental degree of freedom required to specify a classical or quantum micro-state is inverted consistently under $\mathcal{T}$.

**Implication for reversibility.** Because these microscopic transformations are exact, any failure of macroscopic time reversal cannot be attributed to dynamical asymmetry per se. Instead, irreversibility must emerge from the informational and computational limitations analysed in Sections 4 and 5: the calculation cone for forward prediction is compact, whereas the retrodiction sphere for perfect reversal rapidly inflates and soon exceeds any realistic capacity for data specification or control.

### 6.2 Class I: Fully reversible systems ($\kappa \simeq 1,\ C \simeq 0$)

**Characteristics**

- Governed by time-symmetric equations (Newton, Schrödinger).
- Negligible coupling to the environment; entropy production is statistically reversible.
- Calculation cone and retrodiction sphere coincide: $\Xi_{\mathrm{rev}} \approx \Xi_{\mathrm{fwd}}$.

**Examples**

- *Perfectly elastic collisions* in an ideal gas [30].
- *Isolated qubits* exhibiting high-fidelity Loschmidt echoes [13].

### 6.3 Class II: Computationally fragile systems ($\kappa \gtrsim 10^1$)

Systems that obey time-symmetric dynamics microscopically, yet whose retrodiction sphere expands appreciably under weak perturbations.

**Characteristics**

- Operate far from global equilibrium but maintain partial local control via feedback.
- Forward evolution handled within a modest $\mathcal{C}_{\mathrm{fwd}}$; reversal requires a considerably larger $\mathcal{C}_{\mathrm{rev}}$.
- Highly sensitive to initial conditions; $\partial_\tau \kappa > 0$ but modest.

**Examples**

- *Many-body elastic gases*: microscopic reversals exist, yet the requisite phase-space precision is non-local.
- *Quantum processors with error correction*: coherence preserved only so long as the syndrome record is intact; environmental noise inflates $\Xi_{\mathrm{rev}}$ [26].
- *Biological neural networks*: synaptic plasticity introduces path-dependent modification; reversal demands global read-out of neuronal states [20].

## 6.4 Class III: Structurally irreversible systems ($\kappa \gg 10^6,\ C \gg 1$)

Here deep causal interdependence or dissipative processes embed irreversi bility into the architecture.

**Characteristics**

- High structural complexity; feedback loops of large graph rank.
- Entropy generated and exported continuously; large $\partial_t \Xi$.
- Retrodiction sphere grows super-linearly; practical reversal impossible.

**Examples**

- *Inelastic collisions*, shocks, and viscous dissipation.
- *Radiative systems*: Maxwell retarded potentials require advanced data for reversal.
- *Turbulent flows*: exponential sensitivity $\kappa \propto e^{\lambda t}$.
- *Digital computers*: logical erasure incurs Landauer heat [24].
- *Living metabolism*: irreversible biochemical cycles.
- *Spiral galaxies*: density-wave dissipation prevents the re-formation of the original arm pattern under time inversion.

## 6.5 Class IV: CPT-invariant systems (symmetry after full CPT, $\kappa$ undefined)

The combined operation of charge conjugation, parity inversion, and time reversal leaves the fundamental equations invariant; a CPT-conjugated system is not, however, identical to a simple time-reverse.

**Characteristics**

- Microscopic fields transform as prescribed by the CPT theorem.
- Global quantities (stress-energy, causal structure) are preserved.
- Practical realisation would require converting matter to antimatter and mirroring spatial geometry.

**Examples**

- *Relativistic quantum field theories* obeying CPT [31].
- *Thought-experiment universes* evolving backward in CPT sense.

### 6.6 Summary

Microscopic time symmetry ensures that linear momentum, angular momentum, and spin invert cleanly under $\mathcal{T}$; nevertheless macroscopic reversibility depends on the informational cost of specifying past states. Class I systems, with $\kappa \approx 1$, retain full reversibility because their causal information remains locally confined. Classes II and III exhibit increasing $\kappa$ owing to expanded retrodiction spheres and structural complexity, rendering reversal progressively infeasible. Class IV illustrates the distinction between fundamental symmetry (CPT) and computational attainability: invariance exists in principle, yet the required transformations are beyond practical reach. In toto, this taxonomy shows how the breakdown of local causal computability transmutes microscopic symmetry into the macroscopic arrow of time.

## 7 Discussion

In conclusion, we now explore the broader implications of our proposed framework and classification scheme, connecting them to fundamental concepts in entropy, time symmetry, and the evolution of complex systems. We examine how the breakdown of time-reversal symmetry, together with the emergence of computational evolution, informs our understanding of both cosmological dynamics and quantum mechanical processes, and we highlight new interpretations, motivations, and avenues for future research.

### 7.1 Implications for Cosmology and Quantum Mechanics

The framework we have developed offers fresh perspectives on the long-standing puzzles associated with the early Universe, in particular the interpretation of its low-entropy initial state and the emergence of the arrow of time. Our approach rests on a reexamination of entropy as a relative quantity and emphasizes the role of dynamical processes, such as continual inflation, inverse inflation mechanisms, and the rise of entropy-resistant structures, in shaping macroscopic irreversibility.

**Low-Entropy Initial State of the Universe:** A central puzzle in cosmology is that the early Universe appears to have begun in an improbably low-entropy state. Conventional accounts postulate an exceptionally ordered Big Bang, yet offer little rationale for this initial condition. In contrast, our framework interprets the early cosmos as *maximally entropic* in its own co-moving frame, its local equilibrium entropy already saturating the bounds set by its microphysical degrees of freedom, while appearing "low" only when measured against the vastly expanded later Universe. Rapid expansion during the inflationary epoch, potentially augmented by inverse inflation dynamics, drove the spatial manifold to outpace the growth of the mass-energy distribution, thereby hiding high local entropy behind super-horizon correlations. This reinterpretation not only dissolves the low-entropy paradox but also naturally aligns with observed large-scale homogeneity and the emergence of entropy gradients that underpin the cosmic arrow of time.

**Breakdown of Time-Reversal Symmetry in Quantum Systems:** In ideally isolated quantum systems, reversible dynamics are demonstrated by protocols such as the Loschmidt echo; however, any coupling to an uncontrolled environment triggers decoherence and irreversible entropy production. Our classification scheme identifies this crossover at the Class II $\rightarrow$ III boundary, where $\kappa \gtrsim 10^2$ signals the practical end of quantum reversibility despite active error correction. This transition is reflected in the rapid expansion of the *retrodiction sphere* relative to the *calculation cone*, quantitatively linking unitary micro-dynamics to the emergent irreversibility seen in NISQ and larger-scale quantum platforms.

**Computational Evolution and the Arrow of Time:** Beyond purely thermodynamic processes, the emergence of sophisticated computational structures plays a critical role in maintaining local order. Biological systems, artificial neural networks, and mechanical timekeepers all evolve mechanisms, such as error correction, heat dissipation, logical redundancy, that confine entropy export to their surroundings. We propose that as the Universe matures, regions featuring high computational sophistication act as localized entropy-resistant pockets, effectively sharpening the arrow of time. This phenomenon can be viewed as an extension of natural selection in the informational domain: systems that optimize local computability while off-loading irreversibility are statistically favored, reinforcing the directional flow of time through an ongoing interplay between structural organization and thermodynamic constraints.

**The Role of CPT Symmetry in Cosmological Evolution:** CPT symmetry assures that the combined operation of charge conjugation, parity inversion, and time reversal leaves the fundamental laws invariant. A fully CPT-conjugated universe would evolve backward in time yet exhibit the same mass-energy distributions, structure formation processes, and entropy dynamics as our own. The two-universe thought experiment demonstrates that both time-directions are dynamically permissible, and that the experiential arrow of time is determined by local $\kappa$–gradients rather than by any built-in temporal bias in the Lagrangian. This reinforces the view that the arrow of time is a relational, emergent phenomenon arising from limited causal access and the growth of retrodiction complexity.

Together, these implications bridge microscopic reversibility and macroscopic irreversibility by integrating nonequilibrium thermodynamics, quantum information theory, and computational evolution into a single coherent narrative. They suggest that the arrow of time emerges from multiple, interlocking mechanisms whose combined effect enforces temporal directionality across all scales.

## 7.2 Speculative Outlook: A Universal Computational Principle of Evolution

Motivated by the recurrent appearance of low-$\kappa$ subsystems at successive levels of complexity, we propose a conjectural, hierarchical principle of "computational selection." Although speculative, this outlook seeks to unify the emergence of complexity from cosmic to technological scales:

1. **Cosmic Instantiation.** Inflationary regions that minimize $\kappa$ relative to their own microphysics persist longest. In a multiverse setting, this statistical bias may influence the landscape of effective coupling constants and the probability distribution over emergent universes.

2. **Universal Computational Schemes.** Each universe's physical laws may be interpreted as a high-level "program" governing matter–energy dynamics. Mechanisms analogous to black-hole–driven birthing of new spacetimes could instantiate offspring universes with variant rule-sets, forming a branching computational tree.

3. **Emergence of Life and Biological Computation.** In chemically favorable environments, autocatalytic cycles and self-replicators arise. Those systems that maintain low $\kappa$ through localized information processing avoid decoherence and are thus selected by a form of informational natural selection.

4. **Technological and Computational Recursion.** Biological brains give rise to engineered processors and AI systems. These artifacts, in turn, design and improve succeeding generations of computational devices, iterating the arrow-sharpening process and driving $C$ upward while bounding $\Xi_{\rm fwd}$.

5. **Meta-Evolutionary Conjecture.** At each tier, selection favors architectures that minimize the informational cost of forward prediction, i.e. reduce $\kappa$, while relegating irreversibility to the environment. Formalizing this trend as an extremum principle for $\displaystyle\int \kappa^{-1}\, dV\, dt$ or as a variational law in the space of computational architectures remains an open challenge.

Added Motivation and Interpretation:

- The parallel between biological evolution and cosmic structure formation suggests that "life-like" informational dynamics may be a generic feature of any sufficiently complex system, regardless of substrate.
- The inclusion of quantum computation highlights that ultimate limits, such as the speed-of-light barrier, may encode constraints of the universe's own information-processing fabric.

We emphasize that this speculative section is provisional: it serves to integrate our formal constructs with a broader vision of how computational dynamics might underlie complexity throughout the cosmos. Depending on its future elaboration and testing, one may choose either to refine it into a standalone hypothesis or to condense it in subsequent revisions.

### 7.3 Concluding Remarks and Future Work

By replacing absolute entropy with computational naturalness and quantifying irreversibility via the growth of the retrodiction sphere, we have unified early-Universe thermodynamics, quantum decoherence, and the hierarchy of complex systems under a single informational metric. Inflation, decoherence, and computational architecture now appear as diverse yet complementary mechanisms that dilute or expel the information required for perfect time reversal; the macroscopic arrow of time is thus the inevitable shadow cast by limited causal access.

We outline three concrete research programmes to advance this framework:

- **Numerical Cosmology.** Simulate the time-evolution of the naturalness ratio $\kappa(t)$ and the expansion of the retrodiction sphere $\Xi_{\rm rev}(t)$ in lattice models of inflation, reheating, and structure formation, in order to test the information-dilution account of the low-entropy initial condition.
- **Quantum Experiments.** Characterize the Class II $\rightarrow$ III crossover in NISQ and early fault-tolerant quantum devices by monitoring the growth of $\Xi_{\rm rev}$ and tracking the naturalness ratio $\kappa$ under controlled noise injection.

- **Complex-System Statistics.** Quantify how structural complexity $C$, the reversible window $\tau_{\mathrm{rev}}$, and the local naturalness ratio $\kappa$ correlate across biological, technological, and sociotechnical networks, thereby testing the computational-selection hypothesis.

Pursuing these initiatives promises a sharper, quantitatively grounded understanding of why time, though symmetric in the fundamental equations, is experienced, manipulated, and exploited in only one direction.

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